



A Physical, Mechanistic and Fully Coupled Hillslope Evolutionary Model



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Abstract: We present the mathematical and numerical development of a new hillslope hydrology model as well as sample applications. This new model is a distributed, physically and mechanically based hillslope evolutionary model. The model couples the fully two-dimensional hydrodynamic equations for overland flow, Richards equation for infiltration, and a set of sediment detachment and transport equations. This model, based on the fundamental physics of the governing processes of hillslope hydrology is used to test our ability to fully explain the fine scale processes and mechanisms leading to the development of erosion drainage networks. Sample applications are presented to show how the model is capable of capturing the interaction between overland flow, erosion and infiltration at very small scales and of modeling the evolution of hillslope caused by spatially variable erosion caused by small scale variability of the hydraulic and soil properties. We also present analyses with respect to energy expenditure during hillslope evolution.

Introduction: During the past decade, much research on drainage network development has only considered changes in length scales. More specifically length scales generally associated with river and stream networks. The evolution of hillslopes and the drainage networks which develop on them, which are largely responsible for water and sediment delivery to rivers, have yet to be studied within the framework of scaling phenomena. This work focuses on the evolution of hillslopes due to rainfall runoff processes. The idea that hillslope evolution may tend towards a state at which optimality in energy expenditure is achieved is essentially a predictive tool that can be used to estimate hillslope erosional response. However, it is not known whether at any given point in time and space the hillslope system is progressing towards a dynamic equilibrium state, defined by relatively stable characteristics, and to which it will return after a disturbance. Current models of hillslope hydrology and hillslope erosion (e.g., WEPP) separate processes defined by form (e.g., rills and inter-rill areas). If a hillslope is indeed a system progressing towards equilibrium then the rill and inter-rill areas must be intricately tied through the energy expenditure characteristics of the system as a whole as are hillslopes and rivers. As Knighton notes, there is a need to relate the "activity of fluvial processes and the forms that develop to the physical concept of work."

A two-dimensional hillslope model is presented which solves the coupled full hydrodynamic equations for overland flow, Richards equation for infiltration in one-dimension, and a physically based sediment detachment and transport equation. This is the most advanced hillslope model yet to be developed. The use of Richards equation allows continuous simulations of discontinuous rainfall events, and the coupling of the sediment detachment and transport algorithm with the overland flow algorithm allows the modeling of hillslope topographic evolution.

Objectives/Theories/Hypothesis:

- The equilibrium erosion channel network present on a hillslope represents the most efficient network possible to transport the water and sediment input determined by an "effective" rainfall rate.
- For a hillslope system with a variable rainfall input, the "effective" rainfall rate is the rainfall rate at which the most sediment is transported with respect to when and for how long it occurs.
- Drainage density (D) is largely dependent on the minimum energy required to overcome physical restraints (p) and cause soil detachment by overland flow, and on the energy input to the system by the "effective" rainfall rate (r_e). $D = f(r_e, p)$
- The physical restraints imposed on a system include soil particle size and distribution, soil compaction, soil hydraulic characteristics and hillslope topography.
- Hillslope drainage network development is governed by the same scaling processes as their larger river counterparts (e.g., energy dissipation).

Model Description

Fully coupled 2-D overland flow and erosion model with spatially variable infiltration and microtopography. The hydraulic component developed of Fiedler and Ramirez² (2000) based on MacCormack finite difference scheme. The erosion component is transport limited at very low unit discharge and supply limited at high discharge.

Governing Equations

Hydraulics Component

$$\frac{\partial h}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} - q = 0$$

$$\frac{\partial \tau_x}{\partial x} + \frac{\partial \tau_y}{\partial y} - \frac{\partial (p \tau_x)}{\partial x} - \frac{\partial (p \tau_y)}{\partial y} - g \sin(\alpha_x - \alpha_y) + \frac{\tau_x}{h} q_x = 0$$

$$\frac{\partial \tau_x}{\partial x} + \frac{\partial \tau_y}{\partial y} - \frac{\partial (p \tau_x)}{\partial x} - \frac{\partial (p \tau_y)}{\partial y} - g \sin(\alpha_x - \alpha_y) + \frac{\tau_y}{h} q_y = 0$$

Erosion Component

$$\frac{\partial z}{\partial t} = - \frac{\tau_x}{(1-p) \rho_s} \left(\frac{\partial z}{\partial x} + \frac{\partial q_x}{\partial x} \right) + \frac{\tau_y}{(1-p) \rho_s} \left(\frac{\partial z}{\partial y} + \frac{\partial q_y}{\partial y} \right)$$

Infiltration - Richards Equation

$$\frac{\partial}{\partial t} \left(K(\psi) \frac{\partial \psi}{\partial z} \right) + \frac{\partial K(\psi)}{\partial z} \left(\frac{\partial \psi}{\partial z} \right) = \frac{\partial \psi}{\partial t}$$

Where h is depth of flow, p and q are unit discharges in x and y directions respectively, g is the gravity constant, α_x is the bed slope, α_y is friction slope calculated using Darcy-Weisbach friction factor, q_x is lateral inflow, τ_x is trap efficiency (assumed constant), ρ_s is porosity (assumed constant), ρ_w is total load, q_x is sediment transport as function of unit discharge, ψ is the soil water pressure head, $\theta(\psi)$ is the soil volumetric water content, $K(\psi)$ is the hydraulic conductivity and z denotes a length scale directed parallel with the gravity vector.

The model is an evolutionary model. The rate of change in elevation at a point is,

$$\frac{\partial z}{\partial t} = \sigma_f (QS - (QS)_{crit})$$

$$\frac{\partial z}{\partial t} = \sigma_f (r - \tau_{crit})$$

and

$$\frac{\partial z}{\partial t} = \sigma_f (Q^r - S^r)$$

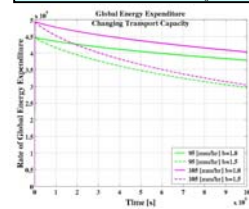
where r is shear stress and τ_{crit} is a shear stress for initiation of sediment transport.

²Fiedler, F.R. and J. A. Ramirez, 2000. A Numerical Method for Hydrodynamic Modeling of Overland Flow. *Int. Jour for Numerical Methods in Fluids*. Vol. 32(2), 219-239.

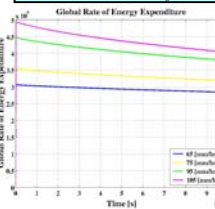
1D Simulations and Results

One-dimensional overland flow tests are conducted to examine the rates of energy expenditure as a function of time and the longitudinal hillslope profile created from an initially smooth hillslope subject to rainfall of constant intensity. The 1-D simulations utilize a grid of 6 X 160 cells with dimensions 6.25 mm in both the x and y directions for a domain 375 mm X 10(10³) mm.

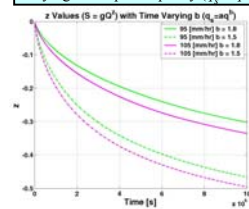
Varying transport capacity ($q_c = q^b$)



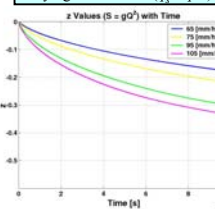
Varying rainfall rate ($q_r = q^{1.8}$)



Varying -transport capacity ($q_c = q^b$)



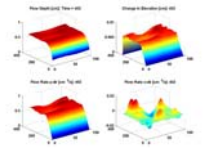
Varying rainfall rate ($q_r = q^{1.8}$)



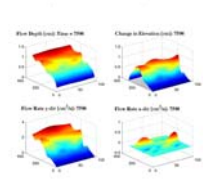
2D Simulation and Results

Initial Conditions are spatially distributed rainfall over a smooth homogenous 9° plane

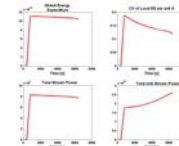
Time = 400 s



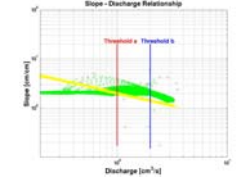
Time = 7600 s



Threshold at $Q = 1.0 \text{ cm}^3/\text{s}$



Green data are raw slope discharge. Yellow is best fit of $S \propto Q^{0.5}$



Threshold at $Q = 2.0 \text{ cm}^3/\text{s}$

Red and blue are thresholds for Definition of flow concentration (see left)

Conclusions:

Results indicate that for a one-dimensional system these equations lead to hillslope longitudinal profiles that minimize the global rate of energy expenditure, the coefficient of variation of the local rate of energy expenditure per unit area, the total unit stream power and the total stream power. The longitudinal profiles developed for the one-dimensional case have slope-area relationships, which approach optimality. The rates at which the energy characteristics are minimized are exponentially related to the rainfall input to the system because the work that can be done by a flow is exponentially related to that flow. A two-dimensional hortonian overland flow model, with detachment and transport-limited erosion, predicts landforms of strong qualitative resemblance to physical slopes in terms of flow paths (when defined using steepest descent criteria). The model does not account well for channelization when beginning with a smooth surface; however, knick points artificially introduced move up slope at a rate consistent with physical studies. Future work will improve resolution of model as well as the erosion component to better represent physical processes on eroding hillslopes. In two-dimensional cases the model shows that the total global rate of energy expenditure and the total stream power approach a minimum throughout hillslope evolution but optimality in unit stream power and the distribution of local energy expenditure per unit area are highly variable and depend critically upon the threshold at which the concentrated flow paths are delineated. Hillslope development occurs at different rates spatially and temporally and therefore do not always approach optimality as a whole but should tend towards optimality, given enough time.